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\[ a^2 \times a^3 \]
\[ F^2 \]
\[ (F^2)^4 \]
\[ \sqrt{a^4} \]
\[ 2e^7 \times 3ef^2 \]
\[ t^2 \div t^2 \]
\[ x^7 \div x^4 \]
\[ 4xy^3 \div 2xy \]
\[ b^1 \]
\[ 5p^5qr \times 6p^2q^6r \]
Expanding single brackets

**Example:**

\[ 4(2a + 3) = 8a + 12 \]

---

**Remember to multiply all the terms inside the bracket by the term immediately in front of the bracket**

**If there is no term in front of the bracket, multiply by 1 or -1**

---

**Expand these brackets and simplify wherever possible:**

1. \(3(a - 4) = 3a - 12\)
2. \(6(2c + 5) = 12c + 30\)
3. \(-2(d + g) = -2d - 2g\)
4. \(c(d + 4) = cd + 4c\)
5. \(-5(2a - 3) = -10a + 15\)
6. \(a(a - 6) = a^2 - 6a\)
7. \(4r(2r + 3) = 8r^2 + 12r\)
8. \(- (4a + 2) = -4a - 2\)
9. \(8 - 2(t + 5) = -2t - 2\)
10. \(2(2a + 4) + 4(3a + 6) = 16a + 32\)
11. \(2p(3p + 2) - 5(2p - 1) = 6p^2 - 6p + 5\)
Expanding double brackets

\[(3a + 4)(2a − 5)\]

\[= 3a(2a − 5) + 4(2a − 5)\]

\[= 6a^2 − 15a + 8a − 20\]

\[= 6a^2 − 7a − 20\]

Split the double brackets into 2 single brackets and then expand each bracket and simplify

“3a lots of 2a − 5 and 4 lots of 2a − 5”

If a single bracket is squared \((a + 5)^2\) change it into double brackets \((a + 5)(a + 5)\)

Expand these brackets and simplify:

1. \((c + 2)(c + 6) = c^2 + 8c + 12\)

5. \((c + 7)^2 = c^2 + 14c + 49\)

2. \((2a + 1)(3a − 4) = 6a^2 − 5a − 4\)

6. \((4g − 1)^2 = 16g^2 − 8g + 1\)

3. \((3a − 4)(5a + 7) = 15a^2 + a − 28\)

4. \((p + 2)(7p − 3) = 7p^2 + 11p − 6\)
If $a = 5$, $b = 6$ and $c = 2$ find the value of:

- $3a = 15$
- $c^2 = 4$
- $4b^2 = 144$
- $ac = 10$
- $ab - 2c = 26$
- $c(b - a) = 2$
- $(3a)^2 = 225$
- $a^2 - 3b = 7$
- $\frac{4bc}{a} = 9.6$
- $(5b^3 - ac)^2 = 1144900$

Now find the value of each of these expressions if $a = -8$, $b = 3.7$ and $c = \frac{2}{3}$.
Solving equations

Solve the following equation to find the value of \( x \):

\[
4x + 17 = 7x - 1
\]

\[
17 = 7x - 4x - 1
\]

\[
17 = 3x - 1
\]

\[
17 + 1 = 3x
\]

\[
18 = 3x
\]

\[
\frac{18}{3} = x
\]

\[
x = 6
\]

\( \leftarrow \text{Take 4x from both sides} \)

\( \leftarrow \text{Add 1 to both sides} \)

\( \leftarrow \text{Divide both sides by 3} \)

Now solve these:
1. \( 2x + 5 = 17 \)
2. \( 5 - x = 2 \)
3. \( 3x + 7 = x + 15 \)
4. \( \frac{4(x + 3)}{5} = 20 \)

Some equations cannot be solved in this way and “Trial and Improvement” methods are required.

Find \( x \) to 1 d.p. if:

\[
x^2 + 3x = 200
\]

<table>
<thead>
<tr>
<th>Try</th>
<th>Calculation</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 10 )</td>
<td>((10 \times 10) + (3 \times 10) = 130)</td>
<td>Too low</td>
</tr>
<tr>
<td>( x = 13 )</td>
<td>((13 \times 13) + (3 \times 13) = 208)</td>
<td>Too high etc.</td>
</tr>
</tbody>
</table>
Solving equations from angle problems

**Find the size of each angle**

Rule involved:
Angles in a quad = \(360^\circ\)

\[4y + 2y + y + 150 = 360\]
\[7y + 150 = 360\]
\[7y = 360 - 150\]
\[7y = 210\]
\[y = 210/7\]
\[y = 30^\circ\]

Angles are: \(30^\circ, 60^\circ, 120^\circ, 150^\circ\)

**Find the value of v**

Rule involved:
“Z” angles are equal

\[4v + 5 = 2v + 39\]
\[4v - 2v + 5 = 39\]
\[2v + 5 = 39\]
\[2v = 39 - 5\]
\[2v = 34\]
\[v = 34/2\]
\[v = 17^\circ\]

Check: \((4 \times 17) + 5 = 73\), \((2 \times 17) + 39 = 73\)
Finding nth term of a simple sequence

Position number (n)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

This sequence is the 2 times table shifted a little

5, 7, 9, 11, 13, 15, ............

Each term is found by the position number times 2 then add another 3. So the rule for the sequence is

\[ n^{th} \text{ term} = 2n + 3 \]

100\(^{th}\) term = \(2 \times 100 + 3 = 203\)

Find the rules of these sequences

1. 1, 3, 5, 7, 9, .... 2n - 1
2. 6, 8, 10, 12, ....... 2n + 4
3. 3, 8, 13, 18, ...... 5n - 2
4. 20, 26, 32, 38, ....... 6n + 14
5. 7, 14, 21, 28, ...... 7n

And these sequences

1. 1, 4, 9, 16, 25, ... \(n^2\)
2. 3, 6, 11, 18, 27 ...... \(n^2 + 2\)
3. 20, 18, 16, 14, ....... -2n + 22
4. 40, 37, 34, 31, ......... -3n + 43
5. 6, 26, 46, 66, ...... 20n - 14
Finding nth term of a more complex sequence

\[ n = 1, 2, 3, 4, 5 \]

\[ 4, 13, 26, 43, 64, \ldots \]

2\textsuperscript{nd} difference is 4 means that the first term is \(2n^2\)

2\textsuperscript{nd} difference:
\[ +9, +13, +17, +21 \]
\[ +4, +4, +4 \]

2\textsuperscript{nd} difference:
\[ +4, +4, +4 \]

2\textsuperscript{nd} difference:
\[ +4, +4, +4 \]

2\textsuperscript{nd} difference:
\[ +4, +4, +4 \]

2\textsuperscript{nd} difference:
\[ +4, +4, +4 \]

This sequence has a rule \(= 3n - 1\)

What's left:
\[ 2, 5, 8, 11, 14, \ldots \]

\[ \text{So the nth term} = 2n^2 + 3n - 1 \]

Find the rule for these sequences:
(a) 10, 23, 44, 73, 110, ...
(b) 0, 17, 44, 81, 128, ...
(c) 3, 7, 17, 33, 55, ...

\[ \rightarrow \text{(a) nth term} = 4n^2 + n + 5 \]

\[ \rightarrow \text{(b) nth term} = 5n^2 + 2n - 7 \]

\[ \rightarrow \text{(c) nth term} = 3n^2 - 5n + 5 \]
Simultaneous equations – 2 linear equations

1. Multiply the equations up until the second unknowns have the same sized number in front of them

\[
\begin{align*}
4a + 3b &= 17 \\
6a - 2b &= 6
\end{align*}
\]

\[
\begin{align*}
8a + 6b &= 34 \\
18a - 6b &= 18
\end{align*}
\]

2. Eliminate the second unknown by combining the 2 equations using either SSS or SDA

\[
\begin{align*}
26a &= 52 \\
a &= \frac{52}{26} \\
a &= 2
\end{align*}
\]

3. Find the second unknown by substituting back into one of the equations

Put \( a = 2 \) into:

\[
4a + 3b = 17
\]

\[
8 + 3b = 17
\]

\[
3b = 17 - 8
\]

\[
3b = 9
\]

\[
b = 3
\]

So the solutions are: \( a = 2 \) and \( b = 3 \)

Now solve:

\[
\begin{align*}
5p + 4q &= 24 \\
2p + 5q &= 13
\end{align*}
\]
Simultaneous equations – 1 linear and 1 quadratic

Sometimes it is better to use a substitution method rather than the elimination method described on the previous slide.

Follow this method closely to solve this pair of simultaneous equations: \( x^2 + y^2 = 25 \) and \( x + y = 7 \)

**Step 1** Rearrange the linear equation: \( x = 7 - y \)

**Step 2** Substitute this into the quadratic: \( (7 - y)^2 + y^2 = 25 \)

**Step 3** Expand brackets, rearrange, factorise and solve:

\[
(7 - y)(7 - y) + y^2 = 25
\]
\[
49 - 14y + y^2 + y^2 = 25
\]
\[
2y^2 - 14y + 49 = 25
\]
\[
2y^2 - 14y + 24 = 0
\]
\[
(2y - 6)(y - 4) = 0
\]
\[
y = 3 \text{ or } y = 4
\]

**Step 4** Substitute back in to find other unknown:

- \( y = 3 \) in \( x + y = 7 \) \( \Rightarrow x = 4 \)
- \( y = 4 \) in \( x + y = 7 \) \( \Rightarrow x = 3 \)
Inequalities can be solved in exactly the same way as equations

14 ≤ 2x – 8
14 + 8 ≤ 2x
22 ≤ 2x
22 ≤ x
2
11 ≤ x

x ≥ 11

Add 8 to both sides
Divide both sides by 2
Remember to turn the sign round as well

The difference is that inequalities can be given as a range of results

Here x can be equal to:
11, 12, 13, 14, 15, …

Or on a scale:

Find the range of solutions for these inequalities:

1. 3x + 1 > 4
   X > 1
   or
   X = 2, 3, 4, 5, 6 …

2. 5x – 3 ≤ 12
   X ≤ 3
   or
   X = 3, 2, 1, 0, -1 …

3. 4x + 7 < x + 13
   X < 2
   or
   X = 1, 0, -1, -2, …

4. -6 ≤ 2x + 2 < 10
   -4 ≤ X < 4
   or
   X = -4, -3, -2, -1, 0, 1, 2, 3
Factorising – common factors

Factorising is basically the reverse of expanding brackets. Instead of removing brackets you are putting them in and placing all the common factors in front.

\[5x^2 + 10xy = 5x(x + 2y)\]

Factorise the following (and check by expanding):

1. \[15 - 3x = 3(5 - x)\]
2. \[2a + 10 = 2(a + 5)\]
3. \[ab - 5a = a(b - 5)\]
4. \[a^2 + 6a = a(a + 6)\]
5. \[8x^2 - 4x = 4x(2x - 1)\]
6. \[10pq + 2p = 2p(5q + 1)\]
7. \[20xy - 16x = 4x(5y - 4)\]
8. \[24ab + 16a^2 = 8a(3b + 2a)\]
9. \[\pi r^2 + 2 \pi r = \pi r(r + 2)\]
10. \[3a^2 - 9a^3 = 3a^2(1 - 3a)\]
Factorising – quadratics

Here the factorising is the reverse of expanding double brackets

To help use a 2 x 2 box

Factorise $x^2 - 9x - 22$

Factor pairs of - 22:
-1, 22
-22, 1
-2, 11
-11, 2

Find the pair which add to give - 9

Answer = $(x + 2)(x - 11)$

Factorising

$\textcolor{red}{x^2 + 4x - 21 = (x + 7)(x - 3)}$

Expanding

Factorise the following:

1. $x^2 + 4x + 3 = (x + 3)(x + 1)$
2. $x^2 - 3x + 2 = (x - 2)(x - 1)$
3. $x^2 + 7x - 30 = (x + 10)(x - 3)$
4. $x^2 - 4x - 12 = (x + 2)(x - 6)$
5. $x^2 + 7x + 10 = (x + 2)(x + 5)$
Factorising - quadratics

When quadratics are more difficult to factorise use this method

Factorise $2x^2 + 5x – 3$

Write out the factor pairs of $–6$ (from 2 multiplied $–3$)

-1, 6
-6, 1
-2, 3
-3, 2

Find the pair which add to give $+5$

(-1, 6)

Rewrite as $2x^2 – 1x + 6x – 3$

Factorise in 2 parts $x(2x – 1) + 3(2x – 1)$

Rewrite as double brackets $(x + 3)(2x – 1)$

Now factorise these:
(a) $25t^2 – 20t + 4$
(b) $4y^2 + 12y + 5$
(c) $g^2 – g – 20$
(d) $6x^2 + 11x – 10$
(e) $8t^4 – 2t^2 – 1$

Answers:
(a) $(5t – 2)(5t – 2)$
(b) $(2y + 1)(2y + 5)$
(c) $(g – 5)(g + 4)$
(d) $(3x – 2)(2x + 5)$
(e) $(4t^2 + 1)(2t^2 – 1)$
### Factorising – grouping and difference of two squares

#### Grouping into pairs

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6ab + 9ad - 2bc - 3cd$</td>
<td>Factorise in 2 parts: $3a(2b + 3d) - c(2b + 3d)$</td>
</tr>
<tr>
<td></td>
<td>Rewrite as double brackets: $(3a - c)(2b + 3d)$</td>
</tr>
</tbody>
</table>

#### Difference of two squares

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x^2 - 25$</td>
<td>Look for 2 square numbers separated by a minus. Simply use the square root of each and a “+” and a “−” to get: $(2x + 5)(2x - 5)$</td>
</tr>
</tbody>
</table>

#### Fully factorise these:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $wx + xz + wy + yz$</td>
<td>$(x + y)(w + z)$</td>
</tr>
<tr>
<td>(b) $2wx - 2xz - wy + yz$</td>
<td>$(2x - y)(w - z)$</td>
</tr>
<tr>
<td>(c) $8fh - 20fi + 6gh - 15gi$</td>
<td>$(4f + 3g)(2h - 5i)$</td>
</tr>
</tbody>
</table>

#### Answers:

<table>
<thead>
<tr>
<th>Expression</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $81x^2 - 1$</td>
<td>$(9x + 1)(9x - 1)$</td>
</tr>
<tr>
<td>(b) $\frac{1}{4} - t^2$</td>
<td>$(\frac{1}{2} + t)(\frac{1}{2} - t)$</td>
</tr>
<tr>
<td>(c) $16y^2 + 64$</td>
<td>$16(y^2 + 4)$</td>
</tr>
</tbody>
</table>
Solving quadratic equations (using factorisation)

Solve this equation:

\[ x^2 + 5x - 14 = 0 \]
\[ (x + 7)(x - 2) = 0 \]
\[ x + 7 = 0 \text{ or } x - 2 = 0 \]
\[ x = -7 \text{ or } x = 2 \]

\[ \leftarrow \text{Factorise first} \]
\[ \leftarrow \text{Now make each bracket equal to zero separately} \]
\[ \leftarrow \text{2 solutions} \]

Solve these:

1. \( 2x^2 + 5x - 3 = 0 \)
   \[ (x + 3)(2x - 1) = 0 \]
   \[ x = -3 \text{ or } x = 1/2 \]

2. \( x^2 - 7x + 10 = 0 \)
   \[ (x - 5)(x - 2) = 0 \]
   \[ x = 5 \text{ or } x = 2 \]

3. \( x^2 + 12x + 35 = 0 \)
   \[ (x + 7)(x + 5) = 0 \]
   \[ x = -7 \text{ or } x = -5 \]

4. \( 25t^2 - 20t + 4 = 0 \)
   \[ (5t - 2)(5t - 2) = 0 \]
   \[ t = 2/5 \]

5. \( x^2 + x - 6 = 0 \)
   \[ (x + 3)(x - 2) = 0 \]
   \[ x = -3 \text{ or } x = 2 \]

5. \( 4x^2 - 64 = 0 \)
   \[ (2x - 8)(2x + 8) = 0 \]
   \[ x = 4 \text{ or } x = -4 \]
Solving quadratic equations (using the formula)

The generalization of a quadratic equation is: \( ax^2 + bx + c = 0 \)

The following formula works out both solutions to any quadratic equation:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Solve \( 6x^2 + 17x + 12 = 0 \) using the quadratic formula

\( a = 6,\ b = 17,\ c = 12 \)

\[
x = \frac{-17 \pm \sqrt{17^2 - 4 \cdot 6 \cdot 12}}{2 \cdot 6}
\]

\[
x = \frac{-17 \pm \sqrt{289 - 288}}{12}
\]

\[
x = \frac{-17 \pm 1}{12}
\]

or

\[
x = \frac{-17 - 1}{12}
\]

\( x = -1.33.. \) or \( x = -1.5 \)

Now solve these:

1. \( 3x^2 + 5x + 1 = 0 \)
2. \( x^2 - x - 10 = 0 \)
3. \( 2x^2 + x - 8 = 0 \)
4. \( 5x^2 + 2x - 1 = 0 \)
5. \( 7x^2 + 12x + 2 = 0 \)
6. \( 5x^2 - 10x + 1 = 0 \)

Answers:

(1) -0.23, -1.43 (2) 3.7, -2.7 (3) 1.77, -2.27 (4) 0.29, -0.69 (5) -0.19, -1.53 (6) 1.89, 0.1
Solving quadratic equations (by completing the square)

Another method for solving quadratics relies on the fact that:

\[(x + a)^2 = x^2 + 2ax + a^2\]  (e.g.  \[(x + 7)^2 = x^2 + 14x + 49\] )

Rearranging: \[x^2 + 2ax = (x + a)^2 – a^2\] (e.g. \[x^2 + 14x = (x + 7)^2 – 49\])

**Example**

Rewrite \[x^2 + 4x – 7\] in the form \[(x + a)^2 – b\]. Hence solve the equation \[x^2 + 4x – 7 = 0\] (1 d.p.)

**Step 1** Write the first two terms \[x^2 + 4x\] as a completed square

\[x^2 + 4x = (x + 2)^2 – 4\]

**Step 2** Now incorporate the third term – 7 to both sides

\[x^2 + 4x – 7 = (x + 2)^2 – 4 – 7\]

\[x^2 + 4x – 7 = (x + 2)^2 – 11\]  (1st part answered)

**Step 3** When \[x^2 + 4x – 7 = 0\] then \[(x + 2)^2 – 11 = 0\]

\[(x + 2)^2 = 11\]

\[x + 2 = \pm \sqrt{11}\]

\[x = \pm \sqrt{11} – 2\]

\[x = 1.3 \quad \text{or} \quad x = -5.3\]
Rearranging formulae

Now rearrange these

1. \( P = 4a + 5 \)
2. \( A = \frac{be}{r} \)
3. \( D = g^2 + c \)
4. \( B = e + \sqrt{h} \)
5. \( E = \frac{u - 4v}{d} \)
6. \( Q = 4cp - st \)

Rearrange the following formula so that \( a \) is the subject

\[ V = u + at \]

\[ a = \frac{V - u}{t} \]

Answers:

1. \( a = \frac{P - 5}{4} \)
2. \( e = Ar \)
3. \( g = \sqrt{D - c} \)
4. \( h = (B - e)^2 \)
5. \( u = d(E + 4v) \)
6. \( p = \frac{Q + st}{4c} \)
### Rearranging formulae

**When the formula has the new subject in two places (or it appears in two places during manipulation) you will need to factorise at some point.**

Now rearrange these:

1. $ab = 3a + 7$
   - $a = \frac{7}{b-3}$

2. $a = \frac{e-h}{e+5}$
   - $e = \frac{-h-5a}{a-1}$

3. $s(t - r) = 2(r - 3)$
   - $r = \frac{st + 6}{2 + s}$

4. $e = \frac{u - 1}{d}$
   - $d = \frac{u}{e + 1}$
Like ordinary fractions, you can only add or subtract algebraic fractions if their denominators are the same.

Show that \( \frac{3}{x+1} + \frac{4}{x} \) can be written as \( \frac{7x + 4}{x(x+1)} \).

\[
\frac{3x}{(x+1)x} + \frac{4(x+1)}{x(x+1)} = \frac{3x}{x(x+1)} + \frac{4x + 4}{x(x+1)} = \frac{3x + 4x + 4}{x(x+1)} = \frac{7x + 4}{x(x+1)}
\]

Multiply the top and bottom of each fraction by the same amount.

Simplify \( \frac{x}{x-1} - \frac{6}{x-4} \).

\[
\frac{x(x-4)}{(x-1)(x-4)} - \frac{6(x-1)}{(x-1)(x-4)} = \frac{x^2 - 4x - 6x + 6}{(x-1)(x-4)} = \frac{x^2 - 10x + 6}{(x-1)(x-4)}
\]
Algebraic fractions – Multiplication and division

Again just use normal fractions principles

Simplify:
\[
\frac{6x}{x^2 + 4x} \div \frac{4x^2}{x^2 + x} = \frac{6x}{x(x + 4)} \times \frac{x(x + 1)}{4x^2} = \frac{3(x + 1)}{2x(x + 4)}
\]

Algebraic fractions – solving equations

Solve: \(\frac{4}{x - 2} + \frac{7}{x + 1} = 2\)

Multiply all by \((x - 2)(x + 1)\)

\[
4(x + 1) + 7(x - 2) = 2(x - 2)(x + 1)
\]

\[
4x + 4 + 7x - 14 = 2(x^2 - 2x + x - 2)
\]

\[
11x - 10 = 2x^2 - 4x + 2x - 4
\]

\[
0 = 2x^2 - 13x + 6
\]

\[
2x^2 - x - 12x + 6 = 0
\]

\[
x(2x - 1) - 6(2x - 1) = 0
\]

\[
(2x - 1)(x - 6) = 0
\]

\[
2x - 1 = 0 \text{ or } x - 6 = 0
\]

\[
x = \frac{1}{2} \text{ or } x = 6
\]
There are four specific types of curved graphs that you may be asked to recognise and draw.

Any curve starting with $x^2$ is “U” shaped.

Any curve starting with $x^3$ is this shape.

Any curve with a number /$x$ is this shape.

If you are asked to draw an accurate curved graph (e.g., $y = x^2 + 3x - 1$), simply substitute $x$ values to find $y$ values and the co-ordinates.
Graphs of $y = mx + c$

In the equation:

$y = mx + c$

$m = \text{the gradient (how far up for every one along)}$

$c = \text{the intercept (where the line crosses the y axis)}$
Graphs of \( y = mx + c \)

Write down the equations of these lines:

Answers:
- \( y = x \)
- \( y = x + 2 \)
- \( y = -x + 1 \)
- \( y = -2x + 2 \)
- \( y = 3x + 1 \)
- \( x = 4 \)
- \( y = -3 \)
Find the region that is not covered by these 3 regions:
- $x \leq -2$
- $y \leq x$
- $y > 3$
Graphing simultaneous equations

Finding co-ordinates for $2y + 6x = 12$
using the “cover up” method:
y = 0 → $2y + 6x = 12$ → $x = 2$ → $(2, 0)$
x = 0 → $2y + 6x = 12$ → $y = 6$ → $(0, 6)$

Finding co-ordinates for $y = 2x + 1$
x = 0 → $y = (2 \times 0) + 1$ → $y = 1$ → $(0, 1)$
x = 1 → $y = (2 \times 1) + 1$ → $y = 3$ → $(1, 3)$
x = 2 → $y = (2 \times 2) + 1$ → $y = 5$ → $(2, 5)$

The co-ordinate of the point where the two graphs cross is $(1, 3)$.
Therefore, the solutions to the simultaneous equations are:
$x = 1$ and $y = 3$
Graphical solutions to equations

If an equation equals 0 then its solutions lie at the points where the graph of the equation crosses the x-axis.

e.g. Solve the following equation graphically:

\[ x^2 + x - 6 = 0 \]

All you do is plot the equation \( y = x^2 + x - 6 \) and find where it crosses the x-axis (the line \( y=0 \)).

There are two solutions to \( x^2 + x - 6 = 0 \):

\[ x = -3 \text{ and } x = 2 \]
Graphical solutions to equations

If the equation does not equal zero:
Draw the graphs for both sides of the equation and where they cross is where the solutions lie

e.g. Solve the following equation graphically:

\[ x^2 - 2x - 11 = 9 - x \]

Plot the following equations and find where they cross:

\[ y = x^2 - 2x - 11 \]
\[ y = 9 - x \]

There are 2 solutions to \[ x^2 - 2x - 11 = 9 - x \]
x = -4 and x = 5

Be prepared to solve 2 simultaneous equations graphically where one is linear (e.g. \( x + y = 7 \)) and the other is a circle (e.g. \( x^2 + y^2 = 25 \))
Expressing laws in symbolic form

In the equation \( y = mx + c \), if \( y \) is plotted against \( x \) the gradient of the line is \( m \) and the intercept on the y-axis is \( c \).

Similarly in the equation \( y = mx^2 + c \), if \( y \) is plotted against \( x^2 \) the gradient of the line is \( m \) and the intercept on the y-axis is \( c \).

And in the equation \( y = \frac{m}{\sqrt{x}} + c \), if \( y \) is plotted against \( \frac{1}{\sqrt{x}} \) the gradient of the line is \( m \) and the intercept on the y-axis is \( c \).
Expressing laws in symbolic form

E.g. y and x are known to be connected by the equation $y = \frac{a}{x} + b$. Find a and b if:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>15</td>
<td>9</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the 1/x values:

<table>
<thead>
<tr>
<th>1/x</th>
<th>1</th>
<th>0.5</th>
<th>0.33</th>
<th>0.25</th>
<th>0.17</th>
</tr>
</thead>
</table>

Plot y against 1/x

So the equation is:

$y = \frac{12}{x} + 3$
Transformation of graphs – Rule 1

The graph of \( y = - f(x) \) is the reflection of the graph \( y = f(x) \) in the x-axis.
The graph of $y = f(-x)$ is the reflection of the graph $y = f(x)$ in the y-axis.
The graph of \( y = f(x) + a \) is the translation of the graph \( y = f(x) \) vertically by vector \( \begin{bmatrix} 0 \\ a \end{bmatrix} \).
The graph of $y = f(x + a)$ is the translation of the graph $y = f(x)$ horizontally by vector $\begin{bmatrix} -a \\ 0 \end{bmatrix}$.
The graph of $y = kf(x)$ is the stretching of the graph $y = f(x)$ vertically by a factor of $k$. 

Transformation of graphs – Rule 5
The graph of $y = f(kx)$ is the stretching of the graph $y = f(x)$ horizontally by a factor of $\frac{1}{k}$. 

**Transformation of graphs – Rule 6**
Straight Distance/Time graphs

- The gradient of each section is the average speed for that part of the journey.
- The horizontal section means the vehicle has stopped.
- The section with the negative gradient shows the return journey and it will have a positive speed but a negative velocity.
- Remember the rule $S = \frac{D}{T}$.

Straight Velocity/Time graphs

- The gradient of each section is the acceleration for that part of the journey.
- The horizontal section means the vehicle is travelling at a constant velocity.
- The sections with a negative gradient show a deceleration.
- The area under the graph is the distance travelled.
- Remember the rule $A = \frac{V}{T}$.